THE MOTION OF A SMALL DROP OF A PARTIALLY WETTING FLUID UNDER THE ACTION OF AN ALTERNATING ELECTRIC CURRENT

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We analyze the motion of a drop of a partially wetting fluid on a horizontal surface for the case in which an alternating electric current is passed through the fluid.

Small volumes of fluid (drops), wetting a solid surface, are formed in a variety of technological processes. In many cases there arises the problem of intensifying the processes of heat and mass transfer within the drops. If the fluid is electrically conductive, an effective means of achieving this intensification may be an alternating electric current under whose action the drops periodically change their shape. As a result, a periodic motion of fluid arises within the drops, and this significantly affects the heat and mass transfer.

Theoretical analysis of the motion of small fluid drops wetting a solid surface is made complicated by the presence of a three-phase contact line (the drop boundaries). The wetting which occurs as the three-phase contact line is shifted exhibits a complex physicochemical nature [1, 2]. As was demonstrated in [3], the hydrodynamic approach to the analysis of this process is impossible because of the incompatibility of the conditions at the free boundary with the conditions of fluid adhesion to the solid surface near the drop boundary (see also [2]). The various solutions of this problem are examined in [4-6]. In particular, it is demonstrated in [6] in conjunction with the results of [7, 8] that consideration of the wedge effect in the hydrodynamic equations makes it possible to state a closed noncontradictory formulation of the problem dealing with the spreading of a partially wetting fluid over a solid surface.

The practical application in these calculations of the recommendations from [4-6] is made difficult because of the high complexity of the wetting process, its dependence on a large number of factors [1, 2, 9], and the absence of detailed data on the wetting process. In this connection, interest has been shown in the situations in which the motion of the three-phase contact line can be neglected. Experimental data [1, 2, 9, 10] show that the velocity of motion for the three-phase contact line is associated with the magnitude of the dynamic contact wetting angle. In this case, under normal conditions, the shifting of the drop boundary is hindered by the fact that the wetting angle may change within significant limits at a nonmoving boundary. With consideration of this phenomenon, neglecting the shift in the drop boundary turns out to be completely valid from the physical standpoint with periodic action of a force of sufficiently high frequency on the drop, when the three-phase contact line fails to "crack" the change in the contact wetting angle.

Let us examine a "flat" drop of an electrically conducting partially wetting fluid (i.e., a fluid such that its contact wetting angle α_m falls within the limits $\pi/2 > \alpha_m > 0$), lying freely on a horizontal plane metal plate (see Fig. 1). A periodic electric current is passed through the system along the uniform z axis. To study drop motion, let us employ the approximation from magnetohydrodynamics [11]. For the sake of brevity, we will not write out the corresponding equations here. Analysis of the magnetohydrodynamic equations and the boundary conditions at the free surface of the drop allows us to define a set of criteria which determine the periodic motion of the fluid: $Re = LU/\nu$ is the Reynolds number, $Re_m = \mu_0 \sigma LU$ is the magnetic Reynolds number, and $S = \delta/L\rho U^2$ is the criterion which determines the role of surface tension. We will define the characteristic velocity as the ratio $U = \Delta L/T$, where T is the characteristic time equal to the period of current oscillation in the system; ΔL is the characteristic magnitude of drop surface displacement, $\Delta L = LP_m/P_0$; $P_m = B_0 j_0 L$ is the

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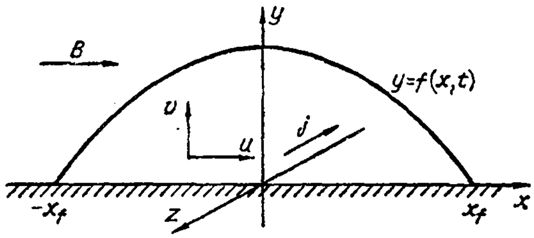

Fig. 1. Drop of an electrically conducting fluid in a magnetic field.

characteristic magnitude of magnetic pressure; $P_0 = \delta/L$ is the characteristic magnitude of the pressure within the drop, generated by surface tension.

For small drops of a good electrically conducting fluid we find the following strong inequalities satisfied: $Re \ll 1$, $Re_m \ll 1$, $S \gg 1$. (For example, for tin at a temperature of 600 K and $L = 10^{-4}$ m, $B_0 = 0.1$ T, $j_0 = 10^7$ A/m², $T = 10^{-2}$ sec, we obtain $Re \approx 0.1$, $Re_m \approx 4 \cdot 10^{-8}$, $S \approx 2 \cdot 10^7$.) For a good wetting fluid in which $\alpha_m \ll 1$, we can assume that the derivatives of all the quantities along and across the drop are also associated by the strong inequality $\partial/\partial y \ll \partial/\partial x$. Satisfaction of these strong inequalities makes it possible significantly to simplify the problem of calculating the motion of the fluid drop, since we can use the noninductive approximation [11] in combination with the approximation of lubrication theory [12]. In this case, in the magnetic pressure we must make allowance only for the interaction of the current flowing within the drop and exhibiting density $j = \{0, 0, j(t)\}$ and the external magnetic field $\mathbf{B} = \{B(t), 0, 0\}$. The equations of the conservation of momentum in this approximation are as follows (because of the small drop dimensions we neglect the force of gravity):

$$\frac{\partial P}{\partial x} - \nu\rho \frac{\partial^2 u}{\partial y^2} = 0, \quad \frac{\partial P}{\partial y} + jB = 0. \tag{1}$$

Equations (1) must be made more complete through the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

the boundary conditions on the solid surface y = 0:

$$u = 0, \quad v = 0 \tag{3}$$

and the boundary conditions on the free surface y = f(x, t):

$$P = -\delta \frac{\partial^2 f}{\partial x^2}, \tag{4}$$

$$\frac{\partial u}{\partial y} = 0, \tag{5}$$

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = v. \tag{6}$$

From Eqs. (1) and conditions (2)-(6) we can determine the projections of velocity in the form:

$$\begin{gathered} u = \frac{1}{\rho\nu}\left(jB\frac{\partial f}{\partial x} - \delta\frac{\partial^3 f}{\partial x^3}\right)\left(\frac{y^2}{2} - fy\right), \\ v = \frac{1}{\rho\nu}\left\{\left(jB\frac{\partial^2 f}{\partial x^2} - \delta\frac{\partial^4 f}{\partial x^4}\right)\left(\frac{y^3}{6} - \frac{fy^2}{2}\right) + \right. \\ \left. + \left(jB\frac{\partial f}{\partial x} - \delta\frac{\partial^3 f}{\partial x^3}\right)\frac{\partial f}{\partial x}\frac{y^2}{2}\right\}. \end{gathered} \tag{7}$$

We will specify the product $jB = j_0B_0(1 + \sin\omega t)$, $\omega = 2\pi/T$. Then, having substituted expression (7) into the kinematic condition (6), we obtain an equation for the determination of the shape of the drop boundary f(x, t):

$$\frac{\partial f}{\partial t}+\frac{1}{\rho\nu}\frac{\partial}{\partial x}\left\{\frac{f^3}{3}\left[\delta\frac{\partial^3 f}{\partial x^3}-j_0B_0(1+\sin\omega t)\frac{\partial f}{\partial x}\right]\right\}=0. \tag{8}$$

It is more convenient to carry out specific calculations in dimensionless variables $x^* = x/L$, $t^* = t/T$, $f^* = f/\alpha L$. Here α is the average value of the contact wetting angle in the motion of the drop. If the vibration amplitude for the drop surface is small, then we may, apparently, assume that $\alpha = \alpha_m$. Let us rewrite Eq. (8) in dimensionless variables:

$$\beta\frac{\partial f}{\partial t}+\frac{\partial}{\partial x}\left\{f^3\left[\frac{\partial^3 f}{\partial x^3}-\varkappa^2\frac{\partial f}{\partial x}(1+\sin 2\pi t)\right]\right\}=0, \tag{9}$$

where $\beta = 3\rho\nu/T\delta\alpha^3$, $\varkappa^2 = j_0B_0L^2/\delta$. Here and in all that follows the asterisk has been dropped from the notation of the dimensionless variables.

For a drop that is symmetrical relative to the plane $x = 0$, Eq. (9) must be considered together with the following conditions at the drop boundary $|x| = x_f$ (see Fig. 1) which are of the form

$$f=0,\quad \left\langle\left|\frac{\partial f}{\partial x}\right|\right\rangle=1,\quad f^3\left(\frac{\partial^3 f}{\partial x^3}-\varkappa^2\frac{\partial f}{\partial x}-\varkappa^2\frac{\partial f}{\partial x}\sin 2\pi t\right)=0, \tag{10}$$

where the angle brackets indicate averaging over time. The second condition in (10) has been written to account for the fact that in the approximation under consideration the slope of the free boundary of the drop and its tangent are indistinguishable. The third condition in (10) physically indicates the absence of a fluid flow rate through the drop boundary $|x| = x_f$. In addition to (10), we must also specify the drop mass, which leads to the condition

$$\int_{-x_f}^{x_f} f\,dx = M, \tag{11}$$

where $M = m/\rho\alpha L^2$.

The invariance of Eq. (9) and conditions (10) and (11) relative to the transformation $x \to -x$ allows us in place of (10) to write the equivalent set of boundary conditions for $x = -x_f$:

$$f=0,\quad \left\langle\frac{\partial f}{\partial x}\right\rangle=1,\quad f^3\left[\frac{\partial^3 f}{\partial x^3}-\varkappa^2\frac{\partial f}{\partial x}(1+\sin 2\pi t)\right]=0 \tag{12}$$

and for $x = 0$

$$\frac{\partial f}{\partial x}=0,\quad \frac{\partial^3 f}{\partial x^3}=0, \tag{13}$$

whose use in the calculations is more convenient, since it reduces the region of integration for Eq. (9).

Let us examine the small vibrations of the free drop boundary near the equilibrium position. We will present the equation for the boundary $y = f(x, t)$ in the form $f(x, t) = f_0(x) + \varepsilon f_1(x, t)$ so that $\langle f(x, t)\rangle = f_0(x)$, where ε is a formal small parameter. Analysis of the boundary-value problem (9), (12), and (13) shows that we can assume that $\varepsilon = \varkappa^2 \ll 1$. Then, for determination of $f_0(x)$ we obtain the problem

$$\begin{gathered}\frac{d^3 f_0}{dx^3}-\varkappa^2\frac{df_0}{dx}=0,\\ f_0=0,\quad \frac{df_0}{dx}=1\ \text{when}\ x=-x_f,\quad \frac{df_0}{dx}=0,\quad \frac{d^3 f_0}{dx^3}=0\ \text{when}\ x=0.\end{gathered} \tag{14}$$

Let us note that the third condition in (12) is used in deriving Eqs. (14), while condition (11) in application to $f_0(x)$ is equivalent to selection of the quantity $x_f > 0$. The solution of problem (14) is written as:

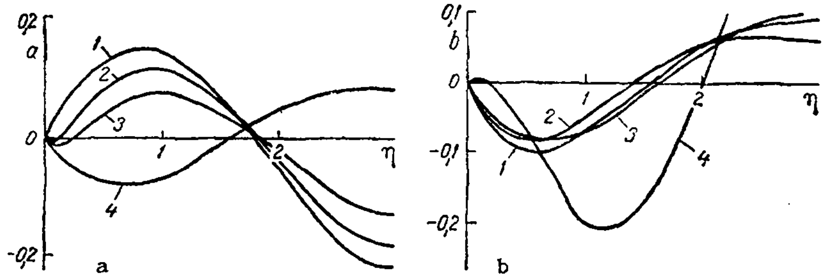

Fig. 2. The functions $a(\eta)$ and $b(\eta)$ for parameter values of $\beta = 3/2\pi$ (1), $5/2\pi$ (2), $7/2\pi$ (3), and $9/2\pi$ (4).

$$f_0 = [\mathrm{ch}(\varkappa x_f) - \mathrm{ch}(\varkappa x)]/\varkappa\, \mathrm{sh}(\varkappa x_f). \tag{15}$$

For determination of the function $f_1(x, t)$, we derive the boundary-value problem

$$\beta \frac{\partial f_1}{\partial t} + \frac{\partial}{\partial x}\left[f_0^3\left(\frac{\partial^3 f_1}{\partial x^3} - \frac{\partial f_0}{\partial x}\sin 2\pi t\right)\right] = 0, \tag{16}$$

$$f_1 = 0, \quad f_0^3\left(\frac{\partial^3 f_1}{\partial x^3} - \varkappa^2 \frac{df_0}{dx}\sin 2\pi t\right) = 0 \quad \text{for} \quad x = -x_f, \tag{17}$$

$$\frac{\partial f_1}{\partial x} = 0, \quad \frac{\partial^3 f_1}{\partial x^3} = 0 \quad \text{for} \quad x = 0, \tag{18}$$

$$\int\limits_{-x_f}^{0} f_1 dx = 0. \tag{19}$$

As we can see, the number of conditions in (17)-(19) exceeds the order of Eq. (16), since the quantity $x_f > 0$ has already been given in the determination of the function $f_0(x)$. The problem can be solved if we note that the second of the conditions in (17) is satisfied automatically, since $f_0|_{x=x_f} = 0$. If we exclude the second of the conditions in (17) from our consideration, we will find the sought boundary-value problem.

Considering the linearity of problem (16)-(19), we will seek the function $f_1(x, t)$ in the form

$$f_1(x, t) = a(x)\sin 2\pi t + b(x)\cos 2\pi t. \tag{20}$$

Having substituted (20) into (16), we obtain a system of ordinary differential equations which, after integration with consideration of the second of the conditions from (18), have the form

$$2\pi\beta\int\limits_0^\eta a\,d\eta + f_0^3\frac{d^3 b}{d\eta^3} = 0, \quad 2\pi\beta\int\limits_0^\eta b\,d\eta + f_0^3\left(\frac{df_0}{d\eta} - \frac{d^3 a}{d\eta^3}\right) = 0, \tag{21}$$

where $\eta = x + x_f$. The boundary conditions for system (21) follow from conditions (17)-(19):

$$a = b = 0 \quad \text{when} \quad \eta = 0, \tag{22}$$

$$\frac{da}{d\eta} = \frac{db}{d\eta} = 0 \quad \text{when} \quad \eta = x_f, \tag{23}$$

$$\int\limits_0^{x_f} a\,d\eta = \int\limits_0^{x_f} b\,d\eta = 0. \tag{24}$$

Equations (21) exhibit a singularity at the point $\eta = 0$, since $f_0|_{\eta=0} = 0$. Analysis of these equations shows that this point is a regular singular point [13] and the solution near this point can be represented in the form of a converging series:

$$a=\ln\eta\sum_{k=2}^{\infty}\alpha_k\eta^k+\sum_{k=1}^{\infty}a_k\eta^k,\quad b=\ln\eta\sum_{k=2}^{\infty}\beta_k\eta^k+\sum_{k=1}^{\infty}b_k\eta^k. \tag{25}$$

Here we have already taken into consideration boundary conditions (22). After substitution of (25) into system (21), we obtain recurrent relationships for the expansion factors (25), which are not presented here because they are too cumbersome. In this case, the coefficients α_2, β_2, a_2, b_2 are determined as conditions (23) and (24) are satisfied. As an example, Fig. 2 shows the results from a calculation of the functions $a(\eta)$ and $b(\eta)$ according to formulas (25), in which we took into consideration the first 60 terms, for the values of the parameters $\varkappa^2 = 0.1$; $x_f = 3$. As we can see, with a change in the parameter β the functions $a(\eta)$ and $b(\eta)$ do not change monotonically.

The functions $a(\eta)$ and $b(\eta)$ allow us to determine the projections of the velocities $u(x, y, t)$ and $v(x, y, t)$ from (7). The next stage is the calculation of the characteristics for the processes of convective heat and mass transfer, but this problem goes beyond the scope of this article.

In conclusion, let us take note of the fact that Eq. (9), the analysis of which was the whole point of the problem, is a quasilinear degenerating equation of the parabolic type. A great number of papers (see reviews [14, 15]) have been devoted to the study of various physical objects described by equations of this type; however, in these reviews we find consideration given only to second-order equations. A rather complete qualitative analysis of the solutions for Eq. (9) is presently not available.

NOTATION

μ_0, magnetic constant; ρ, density; σ, electrical conductivity; ν, kinematic viscosity; δ, surface tension of the fluid; B_0, j_0, oscillation amplitudes for the external magnetic field and for the current density which brings it about; L, characteristic drop dimension; P, pressure; u, v, projections of velocity; x, y, t, coordinates and time; m, mass of the drop.

LITERATURE CITED

1. B. V. Deryagin and N. V. Churaev, Wetting Films [in Russian], Moscow (1984).
2. V. E. B. Dussan, Annu. Rev. Fluid Mech., 11, 371-400 (1979).
3. V. V. Pukhnachev and V. A. Solonnikov, Prikl. Mat. Mekh., 46, No. 6, 961-971 (1982).
4. L. M. Hocking, J. Fluid Mech., 79, Pt. 2, 209-229 (1977).
5. O. V. Voinov, Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 92-96 (1977).
6. K. B. Pavlov, A. S. Romanov, and A. P. Shakhorin, Numerical Methods for the Mechanics of Continuous Media [in Russian], Vol. 17, No. 3 (1986), pp. 132-138.
7. C. A. Miller and E. Ruckenstein, J. Colloid Interface Sci., 48, 368-373 (1974).
8. E. Ruckenstein and C. S. Dunn, J. Colloid Interface Sci., 59, No. 1, 135-138 (1977).
9. B. V. Deryagin, N. V. Churaev, and V. M. Muller, Surface Forces [in Russian], Moscow (1985).
10. P. J. De Gens, Usp. Fiz. Nauk, 151, No. 4, 619-981 (1987).
11. A. B. Vatazhin, G. A. Lyubimov, and S. A. Regirer, Magnetohydrodynamic Flows in Channels [in Russian], Moscow (1970).
12. H. Schlichting, Boundary-Layer Theory [Russian translation], Moscow (1974).
13. V. Vazov, Asymptotic Expansions for Solutions of Ordinary Differential Equations [Russian translation], Moscow (1968).
14. K. B. Pavlov, "Processes of transfer in nonclassic media," Preprint, ITPM SO AN SSSR, Novosibirsk (1983).
15. L. K. Martinson, Mathematical Modeling. Processes in Nonlinear Media [in Russian], Moscow (1986), pp. 280-310.